

Simultaneous Linear Equations and Factorisation

Essential information

Simultaneous linear equations

These are of the form

$$ax + by = c$$

$$dx + ey = f$$

when a, b, c, d, e and f are constants.

Expanding brackets

Expressions such as $a(x + b)$ are expanded out.

For example,

$$2(x + 3) = 2x + 6$$

$$x(4 - x) = 4x - x^2$$

$$2x(1 + 3x) = 2x + 6x^2$$

$$a(x + b) = ax + ab$$

- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- If $x^2 + ax + b = (x + p)(x + q)$, then

$$a = p + q \text{ and } b = pq$$

Linear factorisation

Single factors are taken out of an expression.

For example,

$$3x + 9 = 3(x + 3)$$

$$4x^2 + x = x(4x + 1)$$

$$5x^2 + 10x = 5x(x + 2)$$

Quadratic factorisation

Quadratic expressions are factorised into linear factors.

For example,

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

$$x^2 - 4 = (x + 2)(x - 2)$$

$$(2x - 1)(x + 2) = 2x^2 + 3x - 2$$

Simultaneous Linear Equations and Factorisation

Essential information

Also note that, in general, simultaneous linear equations normally have unique solutions, but not when one equation is the multiple of another. For example, when

$$\begin{aligned}x + y &= 1 \\ 2x + 2y &= 2\end{aligned}$$

there is no unique solution. Also when there are two inconsistent equations, for example,

$$\begin{aligned}x + y &= 1 \\ x + y &= 3\end{aligned}$$

there are NO solutions.