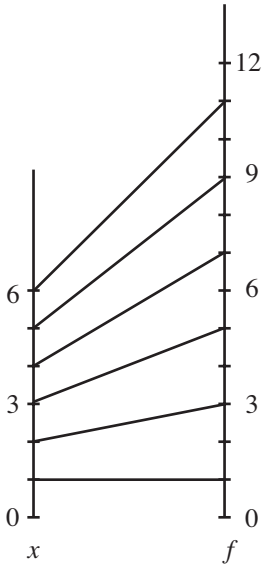
Text *G4 Functions* Answers

G4.1 Functions, Mappings and Domains

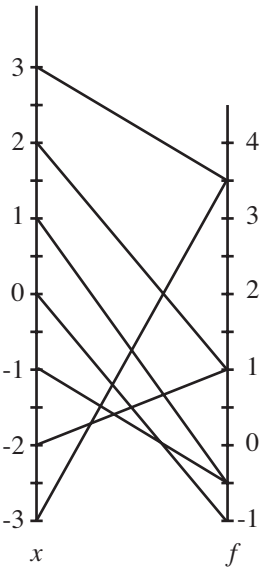
1. (a)



(b) $1 \leq f(x) \leq 11$

(c) Yes

2. (a)



(b) $-1 \leq f(x) \leq 3.5$

(c) No

3. (a) (i) $\frac{1}{3}$ (ii) 0 (iii) -1 (iv) 3 (v) 2

(b) Function not defined at $x = 1$ (as denominator is zero).

Text *G4 Functions* Answers

G4.1

4. (a) $h(-2) = 3$ (b) $x = \pm 4$

5. (a) (i) $f(-2) = \frac{5}{3}$ (ii) $f(0) = 3$ (iii) $f(2) = -1$ (b) $\frac{1}{3}$

6. (a) $f(2) = 6$ (b) $f(-1) = 3$ (c) $f(0) = 2$
   (d) $f(a^2) = a^4 + 2$ (e) $f(1-a) = (1-a)^2 + 2 = a^2 - 2a + 3$

7. (a) $g(1) = 1$ (b) $g(-1) = -1$ (c) $g(0.01) = 100$
   (d) $g(a^2) = \frac{1}{a^2}$ (e) $g(1-a) = \frac{1}{(1-a)}$

8. (a) (i) -3 (ii) 0 (iii) 1 (iv) 0 (v) -3
   (b) $x = \pm 3$

9. (a) $5^2 - 5 \times 3 = 10$
   (b) $-g^2 - 4g + 3 = 0 \Rightarrow g = 1, 3$

G4.2 Composite Functions

1. (a) $g(2) = 3$ (b) $f(-2) = -5$ (c) $fg(2) = \frac{5}{6}$ (d) $gf(-2) = -11$

2. (a) (i) $x^3 - 1$ (ii) $(x-1)^3$
   (b) Two solutions; $x = 0$ or $x = 1$

3. (a) $fg(x) = \frac{1}{x+1}$ (b) $gf(x) = \frac{1}{x} + 1$

4. $fg(x) = 1 + \frac{1}{x^2}$, $gf(x) = \left(1 + \frac{1}{x}\right)^2$; $h(1) = 2$, $h(-1) = -2$

5. (a) $fg(x) = x$, $gf(x) = x$

6. $fg(x) = acx + ad + b$
   $gf(x) = acx + cb + d$
   and result follows

Text *G4 Functions* Answers

G4.3 Inverse Functions

1. (a) $x - 2 \ (\text{all } x)$ (b) $\frac{1}{4}(x + 2) \ (\text{all } x)$ (c) $x \ (\text{all } x)$

   (d) $\frac{3}{x} \ (x \neq 0)$ (e) $\frac{1}{x} - 2 \ (x \neq 0)$

2. $f^{-1}(x) = \frac{3 + x}{4}$

3. (a) 18 (b) 2 (c) 7

4. (a) $g(3) = 7$ (b) $fg(2) = \frac{5}{6}$ (c) $f^{-1}(x) = \frac{3x + 1}{2 - x}$

5. (a) (i) 0 (ii) -3 (iii) $\frac{1}{4}$

   (b) $f^{-1}(x) = \frac{1}{x + 4}$; $f^{-1}(0) = \frac{1}{4}$; $f^{-1}(-3) = 1$

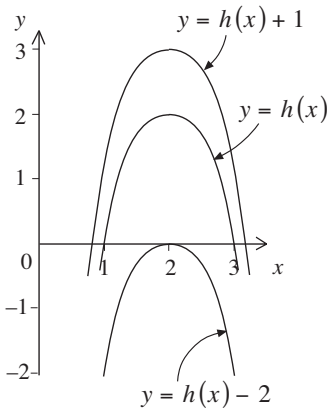
G4.4 Transformations of Graphs of Functions

1. A : $y = f(x + 2)$

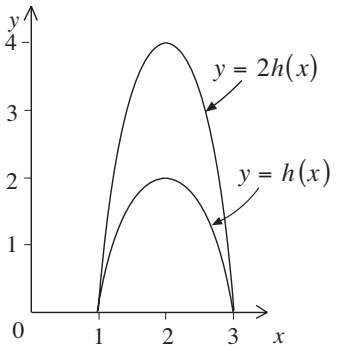
   B : $y = f(x - 3)$

   C : $y = f(x - 5)$

2. (a)



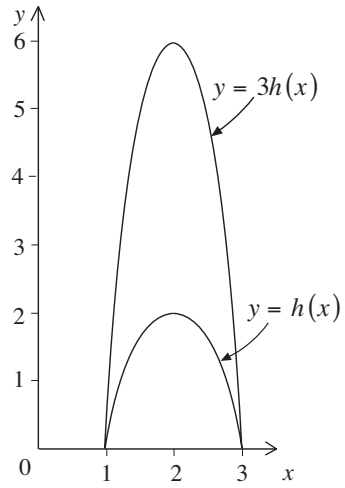
(b)



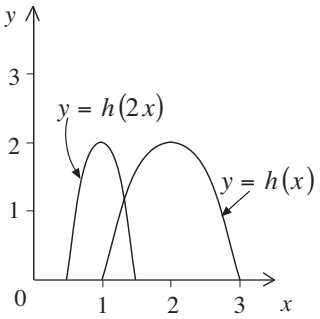
Text *G4 Functions* Answers

G4.4

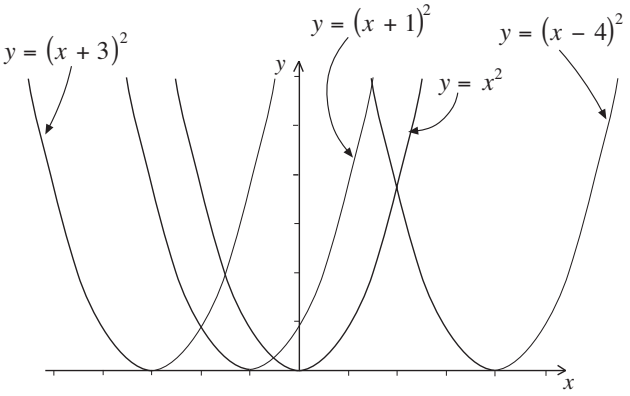
(c)



(d)

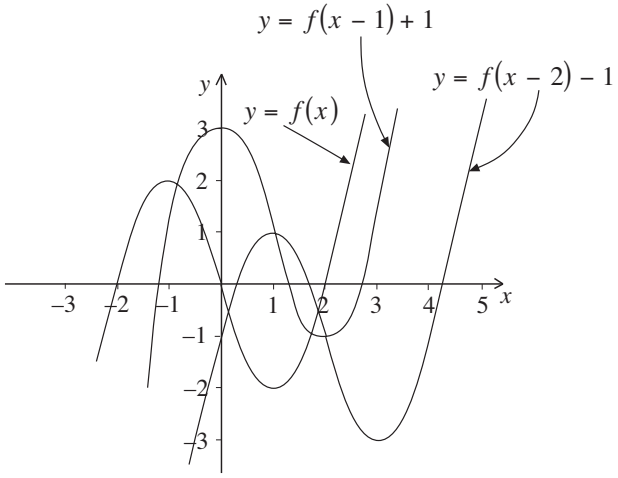


3.



4. (a) Move $x = f(x)$ 2 units along the positive x-axis, and then 2 units up the y-axis.

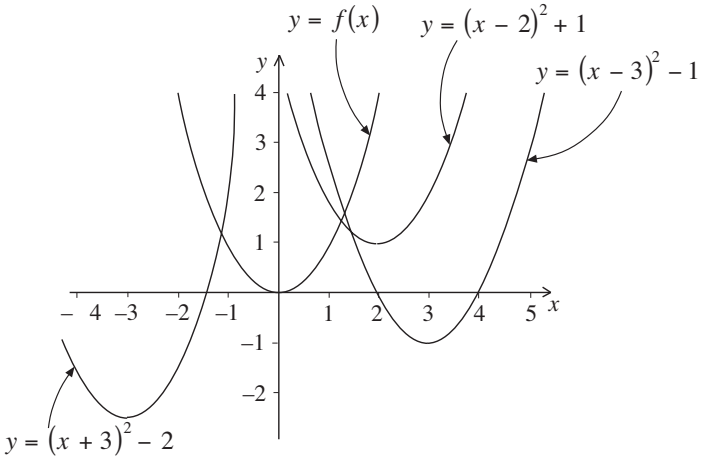
(b)



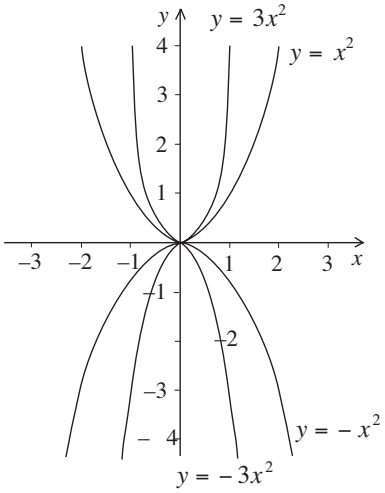
Text *G4 Functions* Answers

G4.4

5.

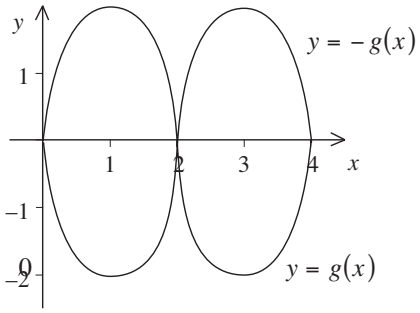


6.

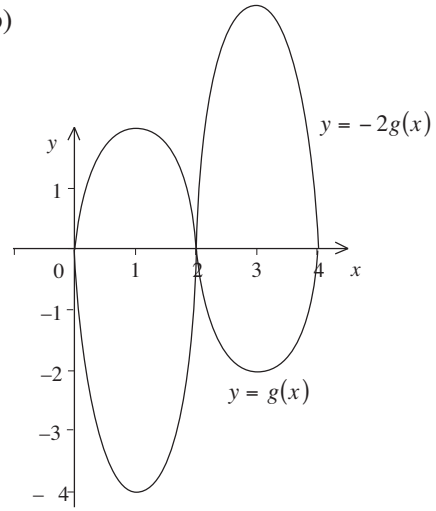


(graphs are reflections of each other)

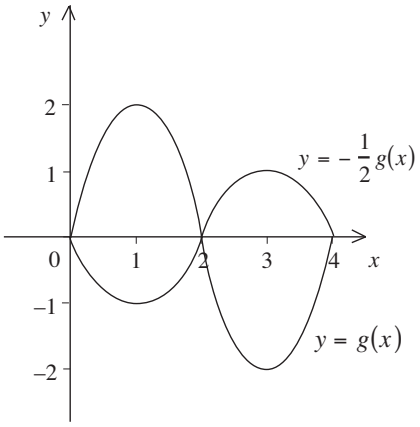
7. (a)



(b)



(c)



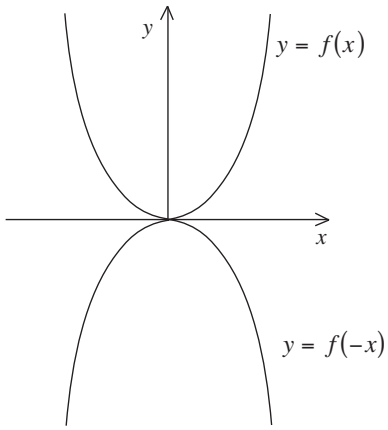
Text *G4 Functions* Answers

G4.4

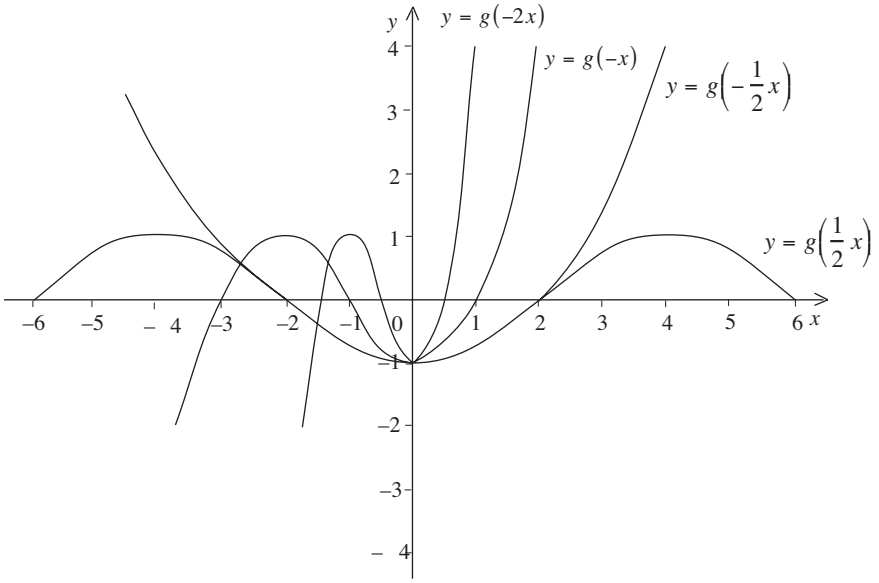
8. (a) $y = f(x)$ and $y = -f(x)$

(b) $y = f(x-1)$ and $y = -f(x-1)+1$

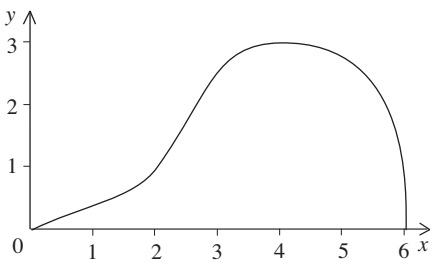
9. (a)



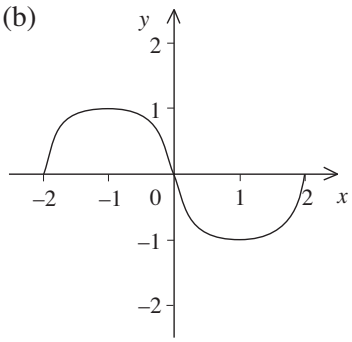
(b)



10. (a)



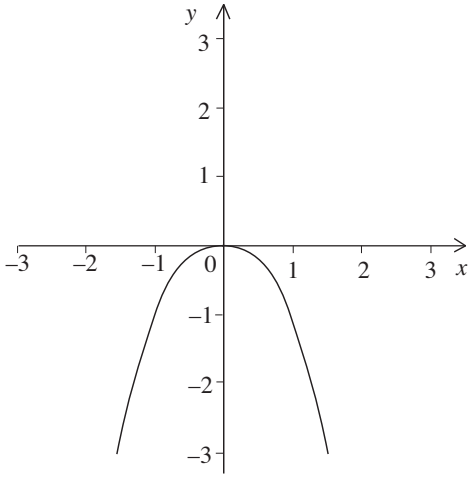
(b)



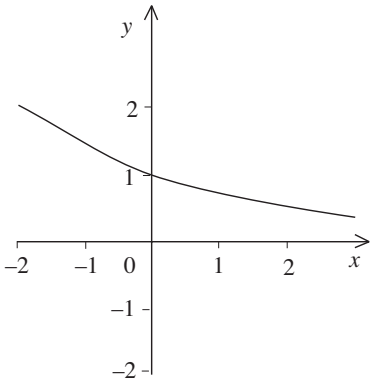
Text *G4 Functions* Answers

G4.4

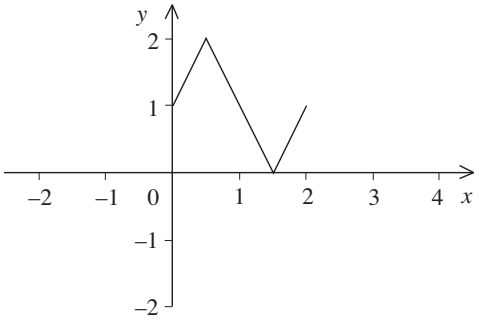
(c)



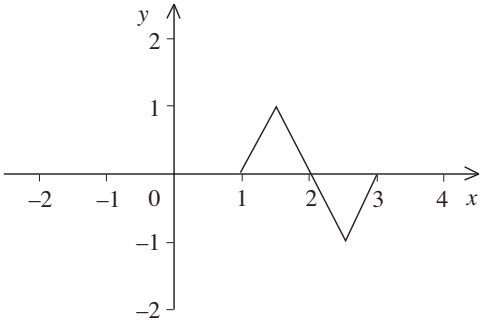
(d)



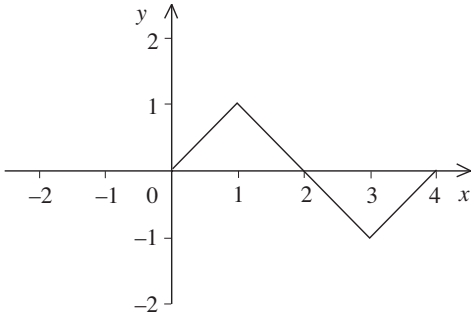
11. (a)



(b)



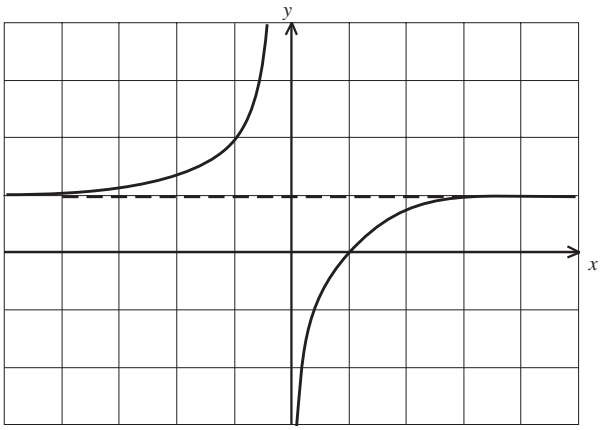
(c)



Text *G4 Functions* Answers

G4.4

12. (a)



(b)